# ADVANGED BANK MANAGEMENT 

## ADVANCED BANK MANAGEMENT

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# INDIAN INSTITUTE OF BANKING \& FINANCE <br> (ISO 9001:2015 Certified) <br> Kohinoor City, Commercial-II, Tower-1, 2nd \& 3rd Floor, <br> Kirol Road, Off-L.B.S. Marg, Kurla-West, <br> Mumbai-400070 

## Established on 30th April 1928

## MISSION

- To develop professionally qualified and competent bankers and finance professionals primarily through a process of education, training, examination, consultancy/counselling and continuing professional development programs.


## VISION

- To be premier Institute for developing and nurturing competent professionals in banking and finance field.


## OBJECTIVES

- To facilitate study of theory and practice of banking and finance.
- To test and certify attainment of competence in the profession of banking and finance.
- To collect, analyse and provide information needed by professionals in banking and finance.
- To promote continuous professional development.
- To promote and undertake research relating to Operations, Products, Instruments, Processes, etc., in banking and finance and to encourage innovation and creativity among finance professionals so that they could face competition and succeed.


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## ADVANCED BANK MANAGEMENT

First Edition (2023)
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## FOREWORD

Formal education will make you a living; self-education will make you a fortune.
-Jim Rohn
The banking sector, currently, is experiencing a transformation catalysed by digitalization and information explosion with the customer as the focal point. Besides, competition from NBFCs, FinTechs, changing business models, growing importance of risk and compliance, along with disruptive technologies, have contributed to this radical shift. Such an ever-evolving ecosystem requires strategic agility and constant upgradation of skill levels on the part of the Banking \& Finance professionals to chart a clear pathway for their professional development.

The mission of the Indian Institute of Banking \& Finance is to develop professionally qualified and competent bankers and finance executives primarily through a process of education, training, examination, counseling and continuing professional development programs. In line with the Mission, the Institute has been offering a bouquet of courses and certifications for capacity building of the banking personnel.

The flagship courses/examinations offered by the Institute are the JAIIB, CAIIB and the Diploma in Banking \& Finance (DB\&F) which have gained wide recognition among banks and financial institutions. With banking witnessing tectonic shifts, there was an imperative need to revisit the existing syllabi for the flagship courses.

The pivotal point for revising the syllabi was to ensure that, in addition to acquiring basic knowledge, the candidates develop concept-based skills in line with the developments happening in the financial ecosystem and to ensure greater value addition to the flagship courses and to make them more practical and contemporary. This will culminate in creating a rich pool of knowledgeable and competent banking \& finance professionals who are capable of contributing to the sustainable growth of their organizations.

Keeping in view the above objectives, the Institute had constituted a high-level Syllabi Revision Committee comprising of members from public sector banks, private sector banks, co-operative banks and academicians. On the basis of the feedback received from various banks and changes suggested by the Committee, the syllabi of JAIIB \& CAIIB have since been finalized.

The revised CAIIB syllabi will now have four compulsory subjects and one elective subject to be chosen from the five elective subjects. The subjects under the revised CAIIB Syllabi are:

## Compulsory

1. Advanced Bank Management
2. Bank Financial Management
3. Advanced Business \& Financial Management
4. Banking Regulations and Business Laws

## Elective

1. Risk Management
2. Information Technology \& Digital Banking
3. Central Banking
4. Human Resources Management
5. Rural Banking

A new module on Compliance has been introduced in Advanced Bank Management with Compliance, Corporate Governance and Audit becoming the focal point for a resilient banking system. New units covering Risks in Foreign Trade, GIFT-city etc have been added in the Bank Financial Management.
The new subject on Advanced Business \& Financial Management will cover the management principles, the advanced concepts of Financial Management and emerging business solutions including Green Finance and Sustainable Financing.

The subject Banking Regulations \& Business Laws (BRBL) is designed to familiarise the professionals with various laws concerning banking and finance with increased focus on case laws, court judgements covering different areas of banking and finance.

The elective subjects on Risk Management, Information Technology \& Digital Banking and Rural Banking have also been thoroughly revised and will include new units to make the courses more contemporary. Insofar as the electives on Central Banking and Human Resources Management are concerned, new modules on NBFCs and Emerging Scenarios in HRM have been introduced respectively.

As is the practice followed by the Institute, a dedicated courseware for every paper/subject is published. The present courseware on Advanced Bank Management has now been authored in line with the revised syllabus for the subject. The book follows the same modular approach adopted by the Institute in the earlier editions/publications.

While the Institute is committed to revise and update the courseware from time to time, the book should, however, not be considered as the only source of information / reading material while preparing for the examinations due to rapid changes being witnessed in all the areas concerning banking \& finance. The students have to keep themselves abreast with the current developments by referring to economic newspapers/journals, articles, books and Government / Regulators' publications / websites etc. Questions will be based on the recent developments related to the syllabus.

Considering that the courseware cannot be published frequently, the Institute will continue the practice of keeping candidates informed about the latest developments by placing important updates/Master Circulars/ Master Directions on its website and through publications like IIBF Vision, Bank Quest, etc.

The courseware has been updated with the help of Subject Matter Experts (SMEs) drawn from respective fields and vetted by practitioners to ensure accuracy and correctness. The Institute acknowledges with gratitude the valuable contributions rendered by the SMEs in updating/vetting the courseware.

We welcome suggestions for improvement of the courseware.

## RECOMMENDED READING

The Institute has prepared comprehensive courseware in the form of study kits to facilitate preparation for the examination without intervention of the teacher. An attempt has been made to cover fully the syllabus prescribed for each module/subject and the presentation of topics may not always be in the same sequence as given in the syllabus.

Candidates are also expected to take note of all the latest developments relating to the subject covered in the syllabus by referring to Financial Papers, Economic Journals, Latest Books and Publications in the subjects concerned.








## ADVANCED BANK MANAGEMENT SYLLABUS

## MODULE A: STATISTICS

Definition of Statistics, Importance \& Limitations \& Data Collection, Classification \& Tabulation Importance of Statistics; Functions of Statistics; Limitation or Demerits of Statistics; Definitions; Collection of Data; Classification and Tabulation; Frequency Distribution

Sampling Techniques
Random Sampling; Sampling Distributions; Sampling from Normal Populations; Sampling from NonNormal Populations; Central Limit Theorem; Finite Population Multiplier
Measures of Central Tendency \& Dispersion, Skewness, Kurtosis
Arithmetic Mean; Combined Arithmetic Mean; Geometric Mean; Harmonic Mean; Median and Quartiles; Mode; Introduction to Measures of Dispersion; Range and Coefficient of Range; Quartile Deviation and Coefficient of Quartile Deviation; Standard Deviation and Coefficient of Variation; Skewness and Kurtosis

## Correlation and Regression

Scatter Diagrams; Correlation; Regression; Standard Error of Estimate

## Time Series

Variations in Time Series; Trend Analysis; Cyclical Variation; Seasonal Variation; Irregular Variation; Forecasting Techniques
Theory of Probability
Mathematical Definition of Probability; Conditional Probability; Random Variable; Probability Distribution of Random Variable; Expectation and Standard Deviation of Random Variable; Binomial Distribution; Poisson Distribution; Normal Distribution; Credit Risk; Value at Risk (VaR); Option Valuation

## Estimation

Estimates; Estimator and Estimates; Point Estimates; Interval Estimates; Interval Estimates and Confidence Intervals; Interval Estimates of the Mean from Large Samples; Interval Estimates of the Proportion from Large Samples
Linear Programming
Graphic Approach; Simplex Method
Simulation
Simulation Exercise; Simulation Methodology

## MODULE B: HUMAN RESOURCE MANAGEMENT

## Fundamentals of Human Resource Management

The Perspective; Relationship between HRM \& HRD and their Structure and Functions; Role of HR Professionals; Strategic HRM; Development of HR Functions in India

## Development of Human Resources

HRD and its Subsystems; Learning and Development - Role and Impact of Learning; Attitude Development; Career Path Planning; Self-Development; Talent Management; Succession Planning

## Human Implications of Organisations

Human Behaviour and Individual Differences; Employees Behaviour at Work; Diversity at Workplace and Gender Issues; Theories of Motivation and their Practical Implications; 'Role’ : Its Concept \& Analysis

## Employees' Feedback and Reward System

Employees' Feedback; Reward and Compensation System

## Performance Management

Appraisal Systems; Performance Review and Feedback; Counselling; Competency Mapping and Assessment of Competencies; Assessment Centres; Behavioural Event Interview (BEI)

## Conflict Management and Negotiation

Conflict: Concept \& Definition; Characteristics of Conflict; Types of Conflicts; Reasons for Conflict; Different Phases of Conflict; Conflict Resolution; Conflict Management; Negotiation Skills for Resolution of Conflicts

## HRM and Information Technology

Role of Information Technology in HRM; HR Information and Database Management; Human Resource Information System (HRIS); Human Resource Management System (HRMS); e-HRM; HR Research; Knowledge Management; Technology in Training; HR Analytics

## MODULE C: CREDIT MANAGEMENT

## Overview of Credit Management

Importance of Credit; Historical Background of Credit in India; Principles of Credit; Types of Borrowers; Types of Credit; Components of Credit Management; Role of RBI Guidelines in Bank's Credit Management

## Analysis of Financial Statements

Which are the Financial Statements; Users of Financial Statements; Basic Concepts Used in Preparation of Financial Statements; Accounting Standards (AS); Legal Position Regarding Financial Statements; Balance Sheet; Profit and Loss Account; Cash Flow Statement; Funds Flow Statement; Projected Financial Statements; Purpose of Analysis of Financial Statements by Bankers; Rearranging the Financial Statements for Analysis; Techniques used in Analysis of Financial Statements; Creative Accounting; Related Party Transactions

## Working Capital Finance

Concept of Working Capital; Working Capital Cycle; Importance of Liquidity Ratios; Methods of Assessment of Bank Finance; Working Capital Finance to Information Technology and Software Industry; Bills/Receivables Finance by the Banks; Guidelines of RBI for Discounting/Rediscounting of Bills by Banks; Trade Receivables Discounting System (TReDS); Non-Fund Based Working Capital Limits; Other Issues Related to Working Capital Finance

Term Loans
Important Points about Term Loans; Deferred Payment Guarantees (DPGs); Difference between Term Loan Appraisal and Project Appraisal; Project Appraisal; Appraisal and Financing of Infrastructure Projects
Credit Delivery and Straight Through Processing
Documentation; Third-Party Guarantees; Charge over Securities; Possession of Security; Disbursal of Loans; Lending under Consortium/Multiple Banking Arrangements; Syndication of Loans; StraightThrough Loan Processing or Credit Underwriting Engines
Credit Control and Monitoring
Importance and Purpose; Available Tools for Credit Monitoring/Loan Review Mechanism (LRM)
Risk Management and Credit Rating
Meaning of Credit Risk; Factors Affecting Credit Risk; Steps taken to Mitigate Credit Risks; Credit Ratings; Internal and External Ratings; Methodology of Credit Rating; Use of Credit Derivatives for Risk Management; RBI guidelines on Credit Risk Management; Credit Information System

## Restructuring/Rehabilitation and Recovery

Credit Default/Stressed Assets/NPAs; Wilful Defaulters; Non-cooperative borrowers; Options Available to Banks for Stressed Assets; RBI Guidelines on Restructuring of Advances by Banks; Available Frameworks for Restructuring of Assets; Sale of Financial Assets

Resolution of Stressed Assets under Insolvency and Bankruptcy Code 2016
Definition of Insolvency and Bankruptcy; To Whom the Code is Applicable; Legal Elements of the Code; Paradigm Shift; Corporate Insolvency Resolution Process; Liquidation process; Pre-packed Insolvency Resolution Process for stressed MSMEs

## MODULE D: COMPLIANCE IN BANKS \& CORPORATE GOVERNANCE

## Compliance Function in Banks

Compliance Policy; Compliance Principles, Process and Procedures; Compliance Programme; Scope of Compliance Function; Role \& Responsibilities of Chief Compliance Officer (CCO)
Compliance Audit
Role of Risk Based Internal Audit and Inspection; Reporting Framework and Monitoring Compliance; Disclosure Requirements; Accounting Standards; Disclosures under Listing Regulations of SEBI

Compliance Governance Structure
Organisational Structure; Responsibility of the Board and Senior Management; Compliance Structure at the Corporate Office; Functional Departments; Compliance Structure at Field Levels; Internal Controls and its Importance

Framework for Identification of Compliance Issues and Compliance Risks
Compliance Issues; Compliance Risk; Inherent Risk and Control Risk; Independent Testing and Effective Audit Programme; Reporting Framework and Monitoring Compliance; Role of Inspection and Audit; Loan Review Mechanism/Credit Audit; What is Good Compliance

Compliance Culture and GRC Framework
How to Create Compliance Culture Across the Organisation; Governance, Risk and Compliance - GRC
Framework; Benefits of an Integrated GRC Approach; Whistle-blower Policy; The Components of a Whistle-blower Policy; Reasons for Compliance Failures

## Compliance Function and Role of Chief Compliance Officer in NBFCs

Framework for Scale Based Regulation for Non-Banking Financial Companies; Transition Path; Framework for Compliance Function and Role of Chief Compliance Officer in Non-Banking Financial Companies in Upper Layer and Middle Layer (NBFC-UL \& NBFC-ML); Broad Contours of Compliance Framework in NBFCs

## Fraud and Vigilance in Banks

Definition of Fraud; Definition of Forgery; Areas in which Frauds are Committed in Banks; Banking and Cyber Frauds; Fraud Reporting and Monitoring System; Vigilance Function in Banks; RBI Guidelines for Private Sector and Foreign Banks on Internal Vigilance

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Unit 1. Definition of Statistics, Importance \& Limitations \& Data Collection, Classification \& Tabulation
Unit 2. Sampling Techniques
Unit 3. Measures of Central Tendency \& Dispersion, Skewness, Kurtosis
Unit 4. Correlation and Regression
Unit 5. Time Series
Unit 6. Theory of Probability
Unit 7. Estimation
Unit 8. Linear Programming
Unit 9. Simulation

### 1.0 OBJECTIVES

After studying this unit, you will be able to:

- Develop an understanding to use the proper methods to collect the data, employ the correct analyses, and effectively present the results.
- Familiarise with Data Management techniques by using Classification, Tabulation and Presentation.
- As Statistics is a crucial process behind how we make discoveries in science, make decisions based on data, and make predictions, collecting correct data and present it properly and making it ready for analysis and interpretation is very important. From this chapter, you will understand correctly getting the data ready to be analysed and interpreted.


### 1.1 INTRODUCTION

The word 'Statistics' has been derived from the Latin word 'statisticum', Italian word 'statistia' and German word 'statistik', each of which means a group of numbers or figures that represent some information of human interest. It was first used by professor Achenwell in 1749 to refer to the subject-matter as a whole. Achenwell defined statistics as the political science of many countries.

In the early years statistics is to be used only by the kings to collect facts about the state, revenue of the state or the people in the state of administrative or political purpose.
Gradually the use of statistics which means data or information has increased and widened. It is now used in almost in all the fields of human knowledge and skills like Business, Commerce, Economics, Social Sciences, Politics, Planning, Medicine and other sciences, physical as well as natural.

In many practical situations in life, we come across different types of data which are needed to be understood, analysed, compared and interpreted correctly. For example, in a college we need to analyse the data of marks obtained, in a hospital we need to analyse the data of number of patients having different diseases, rate of mortality, Different types of data need to be analysed in Economics, Government and Private organisations, Sports and in many other fields. Data mean information, which can be of two types - Qualitative and Quantitative. Statistics means quantitative or numerical data, which can be used for further calculations.

Statistical analysis of data can be comprised of four distinct phases:

1. Collection of data: In this first stage of investigation, numerical data is collected from different published or unpublished sources, primary or secondary.
2. Classification and Tabulation of data: The raw data collected is to be represented properly for further calculations. The raw data is divided into different groups or classes and represented in a form of a table.
3. Analysis of data: Classified and Tabulated data is analysed using different formulas and methods according to purpose of the study or investigation.
4. Interpretation of data: At the final stage, relevant conclusions are drawn after the data is thoroughly analysed

### 1.2 IMPORTANCE OF STATISTICS

Statistics is the subject that teaches how to deal with data, so statistical knowledge helps to use proper methods for collection of data, properly represent the data, use appropriate formula and methods to analyse correctly and effectively get the results and interpret the data. Applications of Statistics is important in every sphere of field - Business and economics, Medical, Sports, Weather forecast, Stock Market, Quality Testing, Government decisions and policies, Banks, Different educational and research organisations, etc.

### 1.2.1 Business and Economics

- In Business, the decision maker takes suitable policies and strategies based on information on production, sale, profit, purchase, finance, etc.
- By using the techniques of time series analysis, the businessman can predict the effect of a large number of variables with a fair degree of accuracy.
- By using 'Bayesian Decision Theory', the businessmen can select the optimal decisions to directly evaluate the payoff for each alternative course of action.
- In Economics, Statistics is used to analyse demand, cost, price, quantity, different laws of demand like elasticity of demand and consumer's maximum satisfaction which is determined on the basis of data pertaining to income and expenditure.


### 1.2.2 Medical

Statistics have extensive application in clinical research and medical field. Clinical research involves investigating proposed medical treatments, assessing the relative benefits of competing therapies, and establishing optimal treatment combinations.

### 1.2.3 Weather Forecast

Statistical methods, like Regression techniques and Time series analysis, are used in weather forecasting.

### 1.2.4 Stock Market

Statistical methods, like Correlation and Regression techniques, Time series analysis are used in forecasting stock prices. Return and Risk Analysis is used in calculation of Market and Personal Portfolios and Mutual Funds.

### 1.2.5 Bank

In banking industry, credit policies are decided based on statistical analysis of profitability, demand deposits, time deposits, credit ratio, number of customers and many other ratios. The credit policies are based on the application of probability theory.

### 1.2.6 Sports

Players use statistics to identify or rectify their mistakes.
A proper understanding of the statistics determines the success of a team or a single athlete.

### 1.3 FUNCTION OF STATISTICS

1. Statistics present the facts in definite form.
2. Statistics simplify complex data.
3. It provides a techniques of comparison.
4. Statistics study the relationship between two or more variables.
5. It helps in formulating policies.
6. It helps in forecasting outcomes.

### 1.4 LIMITATIONS OR DEMERITS OF STATISTICS

1. Statistics do not deal with Individuals

Statistical methods can't be applied for individual values of the observations as for individual observation, there is no point of comparing anything or analysing anything. Statistics is the study of mass data or a group of observations and deals with aggregates of facts.
2. Statistics does not study Qualitative Data

Statistical methods can't be applied for qualitative or non-numerical data. Statistics is the study of only of those facts which are capable of being stated in number or quantity.
3. Statistics give Result only on an Average

Statistical methods are not exact. Generally, when we have large number of observations, it becomes difficult to handle it. A part of the data (sample) is collected for study and draw conclusion from, as a representative for the whole. As a result, the result obtained are not exactly same, had we analysed the whole data. The results are true only on an average in the long run.
4. The results can be biased: The data collection may sometime be biased which will make the whole investigation useless. Generally, this situation arises when data is handled by inexperienced or dishonest person.

### 1.5 DEFINITIONS

## Population

It is the entire collection of observations (person, animal, plant or things which is actually studied by a researcher) from which we may collect data. It is the entire group we are interested in and from which we need to draw conclusions.

## Example

1. If we are studying the weight of adult men in India, the population is the set of weights of all men in India.
2. If we are studying the grade point average of students of Mumbai University, the population is the set of GPA's of all students of Mumbai University.

Sample: Sometimes the population from which we need to draw conclusion is too large to study. At times collecting data from too large a population becomes time-consuming and expensive. To save time and money, generally a part of population is selected for study. A sample is a part (a group of units) of population which is representative of the actual population. By studying the sample, it is expected that valid conclusions are drawn about the whole group.
Example: The population for a study of infant health might be all children born in India in one particular year. The sample might be all babies born on one particular day in that year.

Data can be classified into two types, based on their characteristics. They are:

## 1. Variates

2. Attributes

A characteristic that varies from one individual to another and can be expressed in numerical terms is called variate.
Example: Prices of a given commodity, wages of workers, heights and weights of students in a class, marks of students, etc.
A characteristic that varies from one individual to another but can't be expressed in numerical terms is called an attribute.
Example: Colour of the ball (black, blue, green, etc.), religion of human, etc.
Quantitative or Numerical variables can be further classified as discrete and continuous. A variate which takes discrete or distinct value or in other words can take only a countable and usually finite number of values is called Discrete Variable.
Example: Number of members in a family, Number of accidents, Age in years.
A variate that can take any value within a range (integral/fractional) is called Continuous Variable.
Example: Percentage of marks, Height, Weight.
A parameter is a numerical value or function of the observations of the entire population being studied. A parameter is usually an unknown value that is fixed.
Example: Population mean, population median, population standard deviation, etc.
Since parameter is unknown, it has to be calculated or estimated from a sample. Statistic is used to estimate parameter. A statistic is a quantity or function of the observations of the sample of data. It is used to give information about unknown values in the corresponding population. For example, the sample mean is used to estimate the parameter population mean. Statistic is also called Estimator.

### 1.6 COLLECTION OF DATA

Researchers or investigators need to collect data from respondents. There are two types of data.

### 1.6.1 Primary Data

Primary data is the data which is collected directly or first time by the investigator or researcher from the respondents. Primary data is collected by using the following methods:

Direct Interview Method: A face to face contact is made with the informants or respondents (persons from whom the information is to be obtained) under this method of collecting data. The interviewer asks them questions pertaining to the survey and collects the desired information.

Questionnaires: Questionnaires are survey instruments containing short closed-ended questions (multiple choice) or broad open-ended questions. Questionnaires are used to collect data from a large group of subjects on a specific topic. Currently, many questionnaires are developed and administered online.

## Census and sample survey

In a census, data about all individual units (e.g., people or households) are collected in the population. In a survey, data are only collected for a sub-part of the population; this part is called a sample. These data are then used to estimate the characteristics of the whole population. In this case, it has to be ensured that the sample is representative of the population in question. For example, the proportion of people below the age of 18 or the proportion of women and men in the selected sample of households has to reflect the reality in the total population.

### 1.6.2 Secondary Data

Secondary data are the Second hand information. The data which have already been collected and processed by some agency or persons and is collected for the second time are termed as secondary data. According to M. M. Blair, "Secondary data are those already in existence and which have been collected for some other purpose." Secondary data may be collected from
existing records, different published or unpublished sources, like WHO, UNESCO, LIC, etc., various research and educational organisations, banks and financial places, magazines, internet, etc.

## Distinction between primary and secondary data

1. The data collected for the first time is called Primary data and data collected through some published or unpublished sources is called Secondary data.
2. The primary data in the hands of one person can become secondary for all others.

For example, the population census report is primary for the Registrar General of India and the information from the report is secondary for others.
3. Primary data are original as they are collected first time from the respondents directly or by preparing questionnaires. So they are more accurate than the secondary data. But the collection of primary data requires more money, time and energy than the secondary data. A proper choice between the two forms of information should be made in an enquiry.

### 1.7 CLASSIFICATION AND TABULATION

So, we learned about the different methods of collecting primary and secondary data. The raw data, collected in real situations are arranged randomly, haphazardly and sometimes the data size is very large. Thus, the raw data do not give any clear picture and interpreting and drawing any conclusion becomes very difficult. To make the data understandable, comparable and to locate similarities, the next step is classification of data. The method of arranging data into homogeneous group or classes according to some common characteristics present in the data is called Classification.

Example: The process of sorting letters in a post office, the letters are classified according to the cities and further arranged according to the streets.

Classification condenses the data by removing unimportant details. It enables us to accommodate large number of observations into few classes and study the relationship between several characteristics. Classified data is presented in a more organised way so it is easier to interpret and compare them, which is known as Tabulation.

There are four important bases of classifications:

1. Qualitative Base: Here the data is classified according to some quality or attribute such as sex, religion, literacy, intelligence, etc.
2. Quantitative Base: Here the data is classified according to some quantitative characteristic like height, weight, age, income, marks, etc.
3. Geographical Base: Here the data is classified by geographical regions or location, like states, cities, countries, etc. like population in different states of India.
4. Chronological or Temporal Base: Here the data is classified or arranged by their time of occurrence, such as years, months, weeks, days, etc. This classification is also called Time Series data.

Example: Sales of a company for different years.

## Types of Classification

1. If we classify observed data for a single characteristic, it is known as One-way Classification. Ex: Population can be classified by Religion - Hindu, Muslim, Christians, etc.
2. If we consider two characteristics at a time to classify the observed data, it is known as a Two-way classification. Ex: Population can be classified according to Religion and sex.
3. If we consider more than two characteristics at a time in order to classify the observed data, it is known as Multi-way Classification. Ex: Population can be classified by Religion, sex and literacy.

### 1.8 FREQUENCY DISTRIBUTION

Frequency: If the value of a variable (discrete or continuous) e.g., height, weight, income, etc. occurs twice or more in a given series of observations, then the number of occurrences of the value is termed as the "frequency" of that value.

The way of representing a data in a form of a table consisting of the values of the variable with the corresponding frequencies is called "frequency distribution". So, in other words, Frequency distribution
2. Continuous Frequency Distribution: Variable takes values which are expressed in class intervals within certain limits.

Problem 2: Marks obtained by 20 students in an exam for 50 marks are given below-convert the data into continuous frequency distribution form.
$18,23,28,29,44,28,48,33,32,43,24,29,32,39,49,42,27,33,28,29$.

## Solution

| Marks | Frequency |
| :---: | :---: |
| $15-20$ | 1 |
| $20-25$ | 2 |
| $25-30$ | 7 |
| $30-35$ | 4 |
| $35-40$ | 1 |
| $40-45$ | 3 |
| $45-50$ | 2 |

Problem 3: Following data reveals information about the number of children per family for 25 families. Prepare frequency distribution of number of children (say variable $x$, taking distinct values $0,1,2,3,4)$.

| 3 | 2 | 1 | 1 | 2 |
| :--- | :--- | :--- | :--- | :--- |
| 4 | 0 | 1 | 2 | 3 |
| 1 | 2 | 0 | 4 | 2 |
| 2 | 1 | 2 | 3 | 2 |
| 1 | 3 | 4 | 0 | 1 |

Solution: Frequency distribution of number of children in 25 families

| Number of children | Number of families |
| :---: | :---: |
| 0 | 3 |
| 1 | 7 |
| 2 | 8 |
| 3 | 4 |
| 4 | 3 |
| Total | 25 |

Problem 4: For the following frequency distribution, prepare cumulative frequency distribution of less than and greater than type

| $X$ | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | 5 | 7 | 15 | 11 | 6 | 4 |

Solution

| $X$ | Frequency | Less than of | Greater than of |
| :---: | :---: | :---: | :---: |
| 1 | 5 | 5 | $43+5=48$ |
| 2 | 7 | $5+7=12$ | $36+7=43$ |
| 3 | 15 | $12+15=27$ | $21+15=36$ |
| 4 | 11 | $27+11=38$ | $10+11=21$ |
| 5 | 6 | $38+6=44$ | $4+6=10$ |
| 6 | 4 | $44+4=48$ | 4 |
| Total | 48 |  |  |

Problem 5: Following is the marks of 50 students. Prepare cumulative frequency distribution of both the types. Also find Relative Frequencies.

| Marks | No. of Students |
| :---: | :---: |
| $0-10$ | 7 |
| $10-20$ | 11 |
| $20-30$ | 19 |
| $30-40$ | 7 |
| $40-50$ | 6 |
| Total | 50 |

Solution

| Marks | Cum Frequency | Cum Frequency more than | Relative Frequency |
| :---: | :---: | :---: | :---: |
| $0-10$ | 7 | $43+7=50$ | $7 / 50$ |
| $10-20$ | $7+11=18$ | $32+11=43$ | $11 / 50$ |
| $20-30$ | $18+19=37$ | $13+19=32$ | $19 / 50$ |
| $30-40$ | $37+7=44$ | $6+7=13$ | $7 / 50$ |
| $40-50$ | $44+6=50$ | 6 | $6 / 50$ |

Problem 6: For the following frequency distribution, obtain cumulative frequencies, relative frequencies and relative cumulative frequencies.

| Class Interval | $\mathbf{3 0 - 5 0}$ | $50-70$ | $\mathbf{7 0}-\mathbf{9 0}$ | $\mathbf{9 0}-\mathbf{1 1 0}$ | $\mathbf{1 1 0 - 1 3 0}$ | $\mathbf{1 3 0} \mathbf{- 1 5 0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 8 | 15 | 25 | 16 | 7 | 4 |

Solution

| Class interval | Freq | Cumu. Frequency |  | Relative <br> Frequency | Relative Cumu. frequency |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Less than | More than | Less than | More than |  |
| $30-50$ | 8 | 8 | 75 | 0.11 | 0.11 | 1.00 |
| $50-70$ | 15 | 23 | 67 | 0.20 | 0.31 | 0.89 |
| $70-90$ | 25 | 48 | 52 | 0.33 | 0.64 | 0.69 |
| $90-110$ | 16 | 64 | 27 | 0.22 | 0.86 | 0.36 |
| $110-130$ | 7 | 71 | 11 | 0.09 | 0.95 | 0.14 |
| $130-150$ | 4 | 75 | 4 | 0.05 | $1, .00$ | 0.05 |
| Total | 75 |  |  | 1.00 |  |  |

Problem 7: Represent the information given below with a suitable table.
1400 candidate were medically examined in a fitness test, out of which $21 \%$ were girls. From the doctor's report, it was found that 396 males and 104 females were unfit. $40 \%$ of the remaining males and $60 \%$ of the remaining females were in good health. The rest were declared as temporarily unfit.

Solution: Distribution of candidates according to Male/Female and Fitness

| Fitness $\downarrow$ Sex $\rightarrow$ | Male | Female | Total |
| :---: | :---: | :---: | :---: |
| Good health | 284 | 114 | 398 |
| Temp unfit | 426 | 76 | 502 |
| Unfit | 396 | 104 | 500 |
| Total | 1106 | 294 | 1400 |

Problem 8: Tabulate the information given below.
In 2020, out of total of 3000 workers in a factory, 2300 were skilled workers.
The number of woman employees were 300 out of which 250 were unskilled.
In 2021, the number of skilled workers was 2750 of which 2500 were men. The number of unskilled workers was 760 of which 300 were women.

Solution: Classification of workers according to sex and skill for the year 2020, 2021

| Sex/ <br> Skill <br> Year | Men |  |  |  | Women |  |  |  | Total |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2020 | 2250 | 450 | 2750 | 50 | 250 | 300 | 2300 | 700 | 3000 |  |  |
| 2021 | 2500 | 460 | 2960 | 250 | 300 | 550 | 2750 | 760 | 3510 |  |  |

Problem 9: Out of total number of 2000 candidates interviewed for employment in a company, 628 were from Pune and the rest from Nashik. Amongst the graduate from Pune, 350 were experienced and 80 were unexperienced. While the corresponding figures for undergraduates from Nashik were 615 and

52 respectively. The total number of in experienced candidates from Pune and Nashik were 175 and 192 respectively.

Present the above information in a suitable tabular form.
Solution: Distribution of candidate from Pune and Nashik according to Education and Experience

| Education <br> Experience <br> City | Graduate |  |  |  | Undergraduate |  |  | Total |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Exp | Inexp | Total | Exp | Inexp | Total | Exp | Inexp | Total |  |
| Pune | 350 | 80 | 430 | 103 | 95 | 198 | 453 | 175 | 628 |  |
| Nashik | 565 | 140 | 705 | 615 | 52 | 667 | 1180 | 192 | 1372 |  |
| Total | 915 | 220 | 1135 | 718 | 147 | 865 | 1633 | 367 | 2000 |  |

## Questions

1. The data of some worker's salary are given as $2300,2400,2500,2100,2000,2000,2300,2800$, $3000,2300,2700,2400,2500$. If desired number of class intervals is 10 , class width is
(a) 100
(b) 200
(c) 300
(d) 400
2. The largest and smallest values of a data are 60 and 40 respectively. If desired number of class intervals is 5 , class width is
(a) 25
(b) 20
(c) 4
(d) 5
3. The class intervals where upper and lower limits are also in the class interval are called
(a) Exclusive type
(b) Inclusive type
(c) Discrete type
(d) Continuous type
4. The type of cumulative frequencies where the frequencies are added starting from the highest class to the lowest class are called $\qquad$
(a) Relative Frequency
(b) Percentage Frequency
(c) Less than Cumulative Frequency
(d) Greater than Cumulative Frequency
5. The data classification, which is based on variables, like, demand, supply, height and weight is considered as
(a) Qualitative data
(b) Quantitative data
(c) Time series data
(d) Discrete data
6. The data which is classified or arranged by their time of occurrence, such as years, months, weeks, days, etc., is called
(a) Time series data
(b) Geographical data
(c) Historical data
(d) Both (a) and (c)

## Answers

1. (a);
2. (c);
3. (b);
4. (d);
5. (b);
6. (d)

## UNIT <br> Sampling Techniques



## STRUCTURE

### 2.0. Objectives

2.1. Introduction
2.2. Random Sampling
2.3. Sampling Distributions
2.4. Sampling from Normal Populations
2.5. Sampling from Non-Normal Populations
2.6. Central Limit Theorem
2.7. Finite Population Multiplier

Keywords/Glossary
Questions
Annexure - Probability Table

### 2.0 OBJECTIVES

The objectives of this unit are as follows:

- Learn to take a sample from an entire population and use it to describe the population
- Make sure the samples represent the population.
- Introduce the concepts of sampling distributions
- Understand the tradeoff between costs of larger samples and accuracy
- Introduce experimental design - sampling procedures.
- Estimation - data analysis and interpretation of sample data
- Testing of hypotheses - one-sample data
- Testing of hypotheses - two-sample data


### 2.1 INTRODUCTION

As you know, statistics is a tool used in business and finance. Statistics is an appreciated and maligned tool, depending on how it is used. We need statistical methods to reduce risk and uncertainty and improve our decision-making skills. Decision making in a situation involves collecting data and then using it for the future strategy. Usually. We cannot use the entire data because of the sheer size or numbers. Therefore, we take a sample and test it. For example, if a milk plant processes 1 lakh litres of milk every day, one cannot break open each packet and test the milk for quality. Here we take samples from each batch. If we want to do a market survey, we cannot interview each and every household. We take a representative sample. Thus, sampling becomes an integral tool of the quantitative methods we use. We take a sample, collect data from the sample, and attempt to generalise the whole data results.

Tea tasters at tea auctions are very highly paid employees of tea companies. They sample a small portion of the tea produced from the plantation before the auction. Food products are often tasted before being sold. Before buying Diwali sweets, you may take a small bite before buying it. Obviously, everything cannot be opened and tasted or tested as there would be nothing left to sell. We have to select a sample and test that only. If you want to write a report on why many people are migrating from India to Canada or Australia, contacting each Indian who migrated would be time consuming and expensive. So you would choose a sample and make a report accordingly. Thus, time and size are decisive factors which make it necessary to take business decisions on a sample. If your sample is properly chosen, your report would truly reflect the reasons of the entire population for migration. Of course, the best results would be available in any situation if we collected data from the entire population. Such complete enumeration or census is used in the population census carried out in our country every ten years.

Statisticians use the word population to refer not only to people but to all items that are to be studied. A sample is a part or subset of the population selected to represent the entire group. The word sample is used to describe a portion chosen from the population, in other words.

We can describe samples and populations using mean, median, mode, and standard deviation measures. When these terms describe a sample, they are called statistic and are not from the population but estimated from the sample. When these terms describe a population, they are called parameters.

A statistic is a characteristic of a sample; a parameter is a population characteristic.

Conventionally, statisticians use lower case Roman letters to denote sample statistics and Greek or Capital letters to denote population parameters.
Table 2.1 lists these symbols.

| TABLE 2.1 | Important Statistic Notation and Symbol |  |
| :--- | :--- | :--- |
|  | Population | Sample |
| Definition | Collection of all items <br> Characteristics <br> Symbols | Parameters <br> Size -N <br> Mean $-\mu$ <br> Standard <br> Deviation $-\sigma$ |

## Types of sampling

The process of selecting respondents is known as 'sampling.' The units under study are called sampling units, and the number of units in a sample is called a sample size. There are two methods of selecting samples from populations: non-random or judgement sampling and random or probability sampling. In probability sampling, all the items in the population have a chance of being chosen in the sample. In judgement sampling, personal knowledge or opinions are used to identify the items from the population that are to be included in the sample. A sample selected by judgement sampling is based on someone's experience with the population. An oil drilling company would ask an experienced geologist to test different terrains or land beneath the sea before deciding where to explore for oil. Sometimes a judgement sample is used as a pilot or trial sample to decide how to take a random sample later. If we want to launch a new city newspaper, a pilot test of the paper can be launched at a judgement sample to see the response. But, the rigorous statistical analysis, which can be done with random probability samples, cannot be done with judgement samples. On the other hand, they are more convenient and can be used successfully even if we cannot test their validity. But, if a study uses judgement sampling and loses a significant degree of representativeness, it will have purchased convenience at a high price.

## Biased samples

Suppose the Parliament is debating on the women's bill. You are asked to conduct an opinion survey. Because women are the most affected by the women's bill, you interviewed many women in different cities, towns and rural areas of India. Then you report that an overwhelming 95 per cent are in favour of reservation for women in Parliament.

Sometime later, the government has to take up the issue of Foreign Direct Investment (FDI) in print media. Since newspaper publishers are the most affected, you contact all of them, both national and regional, in India and report that the majority is not in favour of FDI in print media.

In both these cases, you picked a biased sample by choosing people who would have strong feelings on this issue. You have to have sound samples.

A report based on the data collected from such a biased sample would not truly reflect public opinion. If we follow random sampling, it is possible to statistically determine the reliability of the estimates obtained from the sample to avoid such errors.

### 2.2 RANDOM SAMPLING

There are four main types of random sampling.

1. Simple Random Sampling
2. Systematic Sampling
3. Stratified Sampling
4. Cluster Sampling

## Simple Random Sampling

Simple Random Sampling selects samples by methods that allow each possible sample to have an equal probability of being picked and each item in the entire population to be included in the sample.

Suppose we have four teenagers participating in a talk show. We want a sample of two teenagers at a time to participate with the chat show host.

The following table illustrates the possible combinations of samples of teenagers, the probability of each sample being picked and the probability that each teenager will be in a sample.

Teenagers A, B, C, D
Possible samples of two teenagers: $\mathrm{AB}, \mathrm{AC}, \mathrm{AD}, \mathrm{BC}, \mathrm{BD}, \mathrm{CD}$
Probability of drawing these samples of two people is the same as below:

$$
\begin{aligned}
& \mathrm{P}(\mathrm{AB})=1 / 6 \\
& \mathrm{P}(\mathrm{AC})=1 / 6 \\
& \mathrm{P}(\mathrm{AD})=1 / 6 \\
& \mathrm{P}(\mathrm{BC})=1 / 6 \\
& \mathrm{P}(\mathrm{BD})=1 / 6 \\
& \mathrm{P}(\mathrm{CD})=1 / 6
\end{aligned}
$$

You would observe that any one student appears in 3 of the 6 possible samples. Therefore, probability of a particular student in the sample is:

$$
\begin{aligned}
& \mathrm{P}(\mathrm{~A})=1 / 2 \\
& \mathrm{P}(\mathrm{~B})=1 / 2 \\
& \mathrm{P}(\mathrm{C})=1 / 2 \\
& \mathrm{P}(\mathrm{D})=1 / 2
\end{aligned}
$$

The example illustrates a finite population of four teenagers. If we write $\mathrm{A}, \mathrm{B}, \mathrm{C}$, and D on four identical slips of paper, fold the papers, and randomly pick any two, we get a sample. While picking up two paper slips, we may pick up one, keep it away, and pick another from the remaining three. This type is called sampling without replacement.

There is another way of doing it. Suppose after picking the first slip, we note the name on it and put the slip back in the lot, i.e replace the paper slip. Then we draw the second slip. There is a chance that we may draw the same student again. This is called sampling with replacement.
Theoretically, it is possible to have an infinite population. For example, the population of all prime numbers is infinite. Although many populations seem exceedingly large, no truly infinite population of physical objects actually exists. After all, given unlimited resources and time, one can enumerate any finite population. As a practical matter, we will use the term infinite population when we are talking about a population that could not be enumerated in a reasonable period of time. Thus, we use a theoretical concept of infinite population as an approximation of a large finite population.

## How to do Random Sampling?

Suppose there are 100 employees in a company, and we wish to interview a randomly chosen sample of 10. We write the name of each employee on a slip of paper and deposit the slips in a box. After mixing them thoroughly, we draw 10 slips at random. The employees whose names are on these 10 slips, are our random sample.

This method of drawing a sample works well with small groups of people but presents problems with large populations. Also, add to this the problem of the slips of paper not being mixed well.
We can also select a random sample by using random numbers. These numbers can be generated either by a computer programmed to scramble numbers, or by a table of random digits.

The table below shows some random digits. These numbers have been generated by a completely random process. The probability that any one digit from 0 through 9 will appear is the same for each digit, and the probability of one sequence of digits occurring is the same as for any other sequence of the same length.

| 15,819 | 20,685 | 82,621 | 83,748 | 69,662 |
| :--- | :--- | :--- | :--- | :--- |
| 09,281 | 72,950 | 85,961 | 48,980 | 06,840 |
| 41,120 | 48,326 | 18,824 | 54,466 | 94,470 |
| 74,574 | 63,491 | 00,923 | 04,142 | 51,336 |
| 00,995 | 02,727 | 96,703 | 12,671 | 31,976 |
| 59,987 | 93,247 | 72,301 | 34,290 | 69,051 |
| 69,787 | 72,950 | 56,709 | 54,709 | 70,945 |
| 71,642 | 54,358 | 40,316 | 88,897 | 99,907 |
| 35,939 | 34,406 | 11,987 | 23,691 | 70,582 |
| 67,494 | 30,909 | 20,395 | 50,973 | 74,338 |


| 96,974 | 22,756 | 34,574 | 81,235 | 60,089 |
| :--- | :--- | :--- | :--- | :--- |
| 20,079 | 07,000 | 50,890 | 37,689 | 39,622 |
| 45,847 | 76,661 | 55,448 | 14,318 | 74,840 |
| 38,401 | 64,888 | 08,409 | 90,352 | 56,923 |
| 63,151 | 91,208 | 72,286 | 78,612 | 98,120 |
| 01,904 | 02,727 | 18,928 | 53,446 | 01,778 |
| 55,700 | 48,000 | 38,595 | 60,089 | 26,117 |
| 26,926 | 36,478 | 33,822 | 45,786 | 36,984 |
| 31,406 | 67,652 | 15,134 | 53,872 | 87,520 |
| 70,645 | 65,145 | 96,286 | 59,837 | 79,304 |
| 83,832 | 97,712 | 31,209 | 83,209 | 09,007 |
| 11,865 | 68,401 | 98,654 | 43,211 | 11,559 |
| 16,264 | 19,856 | 40,972 | 75,623 | 09,406 |
| 52,203 | 64,287 | 05,963 | 23,673 | 32,329 |
| 49,420 | 22,936 | 42,003 | 32,367 | 15,130 |
| 02,875 | 33,739 | 27,092 | 29,805 | 75,614 |
| 88,094 | 92,125 | 72,709 | 42,514 | 40,618 |
| 86,905 | 64,893 | 74,804 | 69,873 | 40,372 |
| 31,196 | 26,299 | 50,839 | 67,173 | 82,465 |

Let us see how to use this table. We assign each employee a number from 00 to 99 , look up the table above and pick a systematic method of selecting two-digit numbers, like the first two digits. So, we have 15,09 , and so on till we get our 10 numbers.

## Systematic Sampling

In systematic sampling, elements are selected from the population at a consistent level that is measured in time, order, or space. If we wanted to interview every twentieth student on a college campus, we would choose a random starting point in the first twenty names in the student directory and then pick every twentieth name after that.

Systematic sampling differs from simple random sampling in that each element has an equal chance of being selected, but each sample does not have an equal chance of being selected. This would have been the case if, in our earlier example, we had assigned numbers between 00 and 99 to our employees and then began to choose our sample of 10 by picking every tenth number beginning $1,11,21,31$, and so forth. Employees numbered 2, 3, 4 and 5 would have no chance of being selected together.
In systematic sampling, there is a probability of introducing an error into the sampling process. The system chosen may cause a problem. If we want to check the chances of people eating out on different days of the week, choose Friday. There is a higher likelihood of Friday as it is the beginning of the weekend, and we get a higher result.

Systematic sampling has some advantages. Even though systematic sampling may be inappropriate when the elements lie in a sequential pattern, this method may require less time and sometimes results in lower costs than the simple random sample method.

## Stratified Sampling

To use stratified sampling, we divide the population into relatively homogenous groups, called strata. Then we use one of the following two approaches. Either we select at random from each stratum a specified number of elements corresponding to the proportion of that stratum in the population as a whole or, we draw an equal number of elements from each stratum and give weight to the results according to the stratum's proportion of the total population. With either approach, stratified sampling guarantees that every element in the population has a chance of being selected.

## When to use Stratified Sampling

Stratified sampling is appropriate when the population is already divided into groups of different sizes, and we wish to acknowledge this fact. Example - middle class, upper class, lower middle class, etc. or according to age, race, sex or any other stratification. A jeans company may want to study which age group prefers to wear jeans the maximum. Thus the age groups may be 13 to 20,20 to 30,30 to 40 , or 40 plus. The advantages of stratified samples are that when they are properly designed, they more accurately reflect the characteristics of the population from which they are chosen than do other kinds of samples.

## Cluster Sampling

A well-designed cluster sampling procedure can produce a more precise sample at considerably less cost than simple random sampling. In cluster sampling, we divide the population into groups or clusters and then select a random sample of these clusters. We assume that these individual clusters are representative of the population as a whole. Suppose a market Research team is attempting to determine by sampling the average number of television sets per household in a large city. They could use a city map and divide the territory into blocks and then choose a certain number of blocks (clusters) for interviewing. Every household in each of these blocks would be interviewed.

## Comparison of Stratified and Cluster Sampling

The population is divided into well-defined groups with both stratified and cluster sampling. We use stratified sampling when each group has a small variation within itself, but there is wide variation between the groups. We use cluster sampling in the opposite case - when there is considerable variation within each group, but the groups are essentially similar.

## Basis of Statistical Inference: Simple Random Sampling

Systematic sampling, stratified sampling and cluster sampling attempt to approximate simple random sampling. All are methods that have been developed for their precision, economy or physical ease. However, as we do problems, we shall assume that the entire sample we are talking about are data based on simple random sampling. The process of making statistical inferences is based on the principles of
random sampling. Once you understand the basics of random sampling, the same can be extended to other samples with some amendments which are best left to professional statisticians. It is important that you get a grasp of the concepts concerned.

### 2.3 SAMPLING DISTRIBUTIONS

In this section, we presume you are familiar with mathematical concepts such as mean, mode, median, standard deviation, etc. Each sample you draw from a population would have its own mean or measure of central tendency and standard deviation. Thus, the statistics we compute for each sample would vary and be different for each random sample taken.
Let us take an example.
We take a finite population of 5 young boys; A, B, C, D, and E, and collect data about their heights in centimetres. The data is shown in Table 2.2.

| TABLE 2.2 | Sampling Distribution Table |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Boy | A | 'B | $\cdots$. $C$ | D | $E$ |
| height (cm) | 160 | 162 | 164 | 170 | 156 |

Now, if we take samples of size 3 (that is, select 3 boys in each sample), we will get 10 different samples. We list these samples, the corresponding data and their mean in Table 2.3.

| TABLE 2.3 |  | Samples, Their Data and Mean |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Sample | ABC | ABD | ABE | BCD | BCE | ACD | ACE | ADE | BDE | CDE |
| data | 160 | 160 | 160 | 162 | 162 | 160 | 160 | 160 | 162 | 164 |
|  | 162 | 162 | 162 | 164 | 164 | 164 | 164 | 170 | 170 | 170 |
|  | 164 | 170 | 156 | 170 | 156 | 170 | 156 | 156 | 156 | 156 |
| mean | 162 | 164 | 159.33 | 165.33 | 160.66 | 164.66 | 160 | 162 | 162.66 | 163.33 |

From Table 2.3, you can see that sample mean for each sample is different. This collection of different values of the sample mean for samples of size 3 , forms a distribution of sample means. This distribution has a mean. If we add all sample means in Table 2.3, and divide the sum by the number of samples, i.e., 10 , we get 162.397 (say, 162.40)

Normally, we will be dealing with large populations. Hence the number of samples of a particular size is also very large.
Suppose we have to determine the proportion of sugar plants in a plantation affected by pest disease in samples of 100 plants taken from a very large plantation. We have taken a large number of these 100 item
samples. If we plot a probability distribution of the proportions of infested plants in all these samples, we would see a distribution of the sample proportion. (The term proportion here refers to the proportion that is infected.) We could also have a sampling distribution of a proportion.

Sampling distribution is the distribution of all possible values of a statistic from all possible samples of a particular size drawn from the population.

## Describing Sampling Distributions

Any probability distribution (and, therefore, any sampling distribution) can be partially described by its mean and standard deviation. Table 2.4 describes how different sampling distributions can be described.

## TABLE 2.4 Different Sampling Distribution

| S. No. | Population | Sample | Sample Statistic | Sampling Distribution |
| :--- | :--- | :--- | :--- | :--- |
| 1. | Water in a River | 10-one litre con- <br> tainers of water | Mean number of parts of <br> mercury per million parts <br> of water | Sampling distribution of <br> the mean |
| 2. | All professional <br> basketball teams | Groups of 5 <br> players | Median height | Sampling distribution of <br> the median |
| 3. | All parts pro- <br> duced in a manu- <br> facturing process | 50 of each part | Proportion defective | Sampling distribution of <br> the proportion |

Each of the above sampling distributions can be partially described by its mean and standard deviation.

## Concept of Standard Error

The standard deviation of the distribution of the sample means is called the standard error of the mean. Similarly, standard error of the proportion is the standard deviation of the distribution of the sample proportions.

The term standard error is used because it has a very specific connotation. For example, we take various samples to find the average heights of college girls across India and calculate the mean height for each sample. Obviously, there would be some variability in the observed mean. This variability in sampling statistics results from the sampling error due to chance. Thus, the difference between the sample and population means is due to the choice of samples.

Thus, the standard deviation of the sampling distribution of means measures the extent to which the means vary because of a chance error in the sampling process. Thus, the standard deviation of the distribution of a sample statistic is known as the standard error of the statistic.

Thus, a standard error indicates the size of the chance error and the accuracy we are likely to get if we use the sample statistic to estimate a population statistic. Thus, a mean with a smaller standard deviation is a better estimator than one with a higher standard deviation.


## FIGURE 2.2 $\quad$ Probability of Sample Mean Lying between Rs 1900 and Rs 2050

The areas also show the probabilities under the probability curve shown in Fig. 2.2.

### 2.5 SAMPLING FROM NON-NORMAL POPULATIONS

In the preceding section, we stated that when the population is normally distributed, the sampling distribution of the mean is also normal. But, we come across many populations that are not normally distributed. How does the sampling distribution of the mean behave when the population from which the samples are drawn is not normal? An illustration will help us answer this question.

Consider the data in Table 2.5, concerning five motorcycle owners and the life of the Tyres. Because only five people are involved, the population is too small to be approximated by a normal distribution. Let us take all of the possible samples of the owners in groups of three, compute the sample means $(\bar{x})$ list them, and compute the mean of the sampling distribution $\left(\mu_{\bar{x}}\right)$. We have done this in Table 2.5. These calculations show that even in a case in which the population is not normally distributed, $\left(\mu_{\bar{x}}\right)$, the mean of the sampling distribution is still equal to the population mean, $\mu$.

## TABLE 2.5

Experience of Five Motorcycle Owners with Life of Tyres

| Owner | Chetan (C) | Dinesh (D) | Eswar (E) | Feroz (F) | George (G) |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Tyre Life in months | 3 | 3 | 6 | 9 | 15 |

Total life $=36$ months
Mean $=36 / 5=7.2$ months

TABLE 2.6 Calculation of Sample Mean Tyre Life with $n=3$

| Samples of Three | Sample Data | Sample Mean |
| :---: | :---: | :---: |
| EFG | $6+9+15$ | 10 |
| DFG | $3+9+15$ | 9 |
| DEG | $3+6+15$ | 8 |
| DEF | $3+6+9$ | 6 |
| CFG | $6+6+9$ | 7 |
| CEG | $3+6+15$ | 8 |
| CEF | $3+6+9$ | 6 |
| CDF | $3+3+9$ | 5 |
| CDE | $3+3+9$ | 5 |
| CDG | $3+3+15$ | 8 |

Total $=72$ months
Mean $=7.2$ months
Now, look at Fig. 2.3(a), which shows the population distribution of tyre lives for the five motorcycles owners, a distribution that is anything but normal in shape. In Fig. 2.3(b), we show the sampling distribution of the mean for a sample size of three, taking the information from Table 2.6. Notice the difference between the probability distributions in Figs. 2.3(a) and 2.3(b). In Figure 2.3(b), the distribution looks a little more like the bell shape of the normal distribution.

If we repeat this exercise and enlarge the population size to 40 , we could take samples of different sizes. Then plot the sampling distributions of the mean that would occur for the different sizes. This will show quite dramatically how quickly the sampling distribution of the mean approaches normality, regardless of the shape of the population distribution.



FIGURE 2.3 (b)
Distribution of Mean Tyre Life

### 2.6 CENTRAL LIMIT THEOREM

The example in the above Table and the probability distribution in the above graphs tell us many things. First, the mean of the sampling distribution of the mean will equal the population mean regardless of the sample size, even if the population is not normal. As the sample size increases, the sampling distribution of the mean will approach normality, regardless of the shape of the population distribution.

The Central Limit Theorem is the relationship between the shape of the population distribution and the shape of the sampling distribution of the mean. The central limit theorem is perhaps the most important in all statistical inference. It assures us that the sampling distribution of the mean approaches normal as the sample size increases.

1. Actually, a sample does not have to be very large for the sampling distribution of the mean to approach normal.
2. Statisticians use the normal distribution as an approximation to the sampling distribution whenever the sample size is at least 30 , but the sampling distribution of the mean can be nearly normal with samples of even half the size.
3. The significance of the central limit theorem is that it permits us to use sample statistics to make inferences about population parameters without knowing anything about the shape of the frequency distribution of that population

Let's illustrate the use of the central limit theorem. The distribution of annual earnings of all bank tellers with five years' experience is as shown below in Fig. 2.4. This distribution has a mean of Rs. 19,000 and a standard deviation of Rs. 2,000. If we draw a random sample of 30 tellers, what is the probability that their earnings will average more than Rs. 19,750 annually? In Fig. 2.4 we show the sampling distribution of the means that would result and highlight the area representing 'earnings over Rs. 19,750.'

Our first task is to calculate the standard error of the mean from the population standard deviation as follows:


Standard error of the mean

$$
\begin{aligned}
\sigma_{\bar{x}} & =\frac{\sigma}{\sqrt{n}} \\
& =\frac{2000}{\sqrt{30}} \\
& =\frac{2000}{5.477}=\text { Rs. } 365.16
\end{aligned}
$$

Because we are dealing with a sampling distribution, we must now use the equation for z value and the Standard Normal Probability Distribution (App. Table $z=(\bar{x}-\mu) / \sigma_{\bar{x}}$

For

$$
\begin{aligned}
\bar{x} & =\text { Rs. } 19750 ; \\
z & =\frac{(19750-19,000)}{365.16} \\
& =\frac{750}{365.16} \\
& =2.05
\end{aligned}
$$

Annexure Table gives us the probability of 0.4798 for $\mathrm{a} z$ value of 2.05 . We show the corresponding area in Fig. 2.4 as the area between the mean and Rs. 19,750. Since half or 0.5000 of the area under the curve lies between the mean and the right-hand tail, the shaded area must be
0.5000 (Area between the mean and the right-hand tail)

- 0.4798 (Area between the mean and 19,750)
0.0202 (Area between the right-hand tail and 19,750)

Thus, we have determined that there is slightly more than a 2 per cent chance of average earnings being more than Rs. 19,750 annually in a group of 30 tellers.

The central limit theorem is one of the most powerful concepts in statistics, which states that the distribution of sample means tends to be a normal distribution. This is true regardless of the shape of the population distribution from which the samples were taken.
Result 1: If $\mathrm{X} \sim \operatorname{Bin}(\mathrm{n}, \mathrm{p})$, then $\mathrm{Z}=\frac{\mathrm{X}-n p}{\sqrt{n p q}}$ tends to standard Normal Deviation as $\mathrm{n} \rightarrow \infty$
Result 2: If $\mathrm{X} \sim \mathrm{P} \lambda$ ), then $\mathrm{Z}=\frac{\mathrm{X}-\lambda}{\sqrt{\lambda}}$ tends to standard Normal Deviation as sample size $\rightarrow \infty$

## Examples

1. A sample of 25 observations from a normal distribution has a mean of 98.6 and a standard deviation of 17.2 .
(a) What is $\mathrm{P}(92<\bar{x}<102)$ ?
(b) Find the corresponding probability given a sample of 36 .

## Solution:

(a) $N=25, \mu=98.6, \sigma=17.2$,

$$
\begin{aligned}
& \sigma x=\sigma / \sqrt{n}=17.2 / \sqrt{25}=3.44 \\
& P(92<\bar{x}<102)=p\left[(92-98.6) / 3.44<(\bar{x}-\mathrm{m}) / \mathrm{s}_{x}<(102-98.6) / 3.44\right] \\
& =P(-1.72<z<0.99)=0.4573+0.3389=0.7962
\end{aligned}
$$

(b) $n=36, \sigma_{\bar{x}}=\sigma / \sqrt{n}=17.2 / \sqrt{36}=2.87$

$$
\begin{aligned}
& P(92<\bar{x}<102)=p\left[(92-98.6) / 2.87<(\bar{x}-\mu) / \sigma_{x}<(102-98.6) / 2.87\right] \\
& \quad=P(-2.30<z<1.18)=.4893+0.3810=0.8703
\end{aligned}
$$

2. Kamala, an auditor for a large credit card company, knows that, on average, the monthly balance of any given customer is Rs. 112, and the standard deviation is Rs. 56. If Kamala audits 50 randomly selected accounts, what is the probability that the sample average monthly balance is
(a) Below Rs. 100?
(b) Between Rs. 100 and Rs. 130?

Solution: The sample size of 50 is large enough to use the central limit theorem
$\mu=112, \sigma=56, n=50, \sigma_{\bar{x}}=56 / \sqrt{50}=7.920$
(a) $P(\bar{x}<100)=P\left[(\bar{x}-\mu) / \sigma_{\bar{x}}<(100-112) / 7.920\right]$
$=P(z<-1.52)=0.5-0.4357=0.0643$
(b) $P(100<x<130)=P\left[(100-112) / 7.920<(\bar{x}-\mu) / \sigma_{\bar{x}}<(130-112) / 7.920\right]$

$$
=P(-1.52<z<2.27)=0.4357+0.4884=0.9241
$$

3. It has been found that $2 \%$ of the tools produced by a certain machine are defective. What is the probability that in a shipment of 400 tools, $3 \%$ or more defective?
The sample size of 400 is large enough to use the central limit theorem.
Solution: $\mathrm{X} \sim \mathrm{P}(\lambda=n p=0.02 * 400=8)$, then $\mathrm{Z}=\frac{X-\lambda}{\sqrt{\lambda}}$ tends to standard Normal Deviation $3 \%$ of $400=12$
$\mathrm{P}(\mathrm{X}>12)=\mathrm{P}[(\mathrm{x}-8) / \sqrt{8}>(12-8)) / \sqrt{8}=\mathrm{P}([(\mathrm{x}-8) / 2.82>1.43)=0.5-0.4236=0.0764$
4. A coin is tossed 700 times. Using Normal approximation find the probability of getting number of Heads between 280 and 375 .
The sample size of 700 is large enough to use the central limit theorem.
Solution: $\mathrm{X} \sim \operatorname{Bin}(n=700, p=0.5)$, then $\mathrm{Z}=\frac{X-n p}{\sqrt{n p q}}=\frac{X-350}{13.22}$ tends to standard Normal Deviation

$$
\mathrm{P}(280<\mathrm{X}<375)=\mathrm{P}[(280-350) / 13.22<(\mathrm{X}-350) / 13.22<(375-350) / 13.22)]
$$

$=\mathrm{P}[-5.29<\mathrm{Z}<1.89]=0.5+0.4706=0.9706$

## An Important Consideration in Sampling: The Relationship between Sample Size and Standard Error

We saw earlier in this chapter that the standard error, $\sigma_{x}$ is a measure of the dispersion of the sample means around the population mean. If the dispersion decreases (if $\sigma_{x}$ becomes smaller), then the values taken by the sample mean tend to cluster more closely around m. Conversely, if the dispersion increases (if $\sigma_{x}$ becomes larger), the values taken by the sample mean tend to cluster less closely around m . We can think of this relationship this way: As the standard error decreases, the value of any sample mean will probably be closer to the value of the population mean. As the standard error decreases, the precision with which the sample mean can be used to estimate the population mean, increases.

If we refer to Equation, we can see that as $n$ increases, $\mathrm{s}_{x}$ decreases. This happens because, in an Equation, a larger denominator on the right side would produce smaller $s_{x}$ on the left side. Two examples will show this relationship; both assume the same population standard deviation s of 100 .

$$
\text { When } \begin{aligned}
n & =10, \sigma_{\bar{x}}=\sigma / \sqrt{n} \\
& =100 / 3.162=31.63, \text { standard error of the mean. }
\end{aligned}
$$

And when $n=100$ :
Standard error of the mean; $\sigma_{\bar{x}}=100 / \sqrt{100}=10$
What have we shown? As we increased our sample size from 10 to 100 (a tenfold increase), the standard error dropped from 31.63 to 10 , which is only about one-third of its former value. Our examples show that, because $s x$ varies inversely with the square root of $n$, there is diminishing return in sampling.

It is true that sampling more items will decrease the standard error, but this benefit may not be worth the cost. It seldom pays to take excessively large samples. Managers should always assess both the worth
and the cost of the additional precision they will obtain, from a larger sample, before they commit resources to take it.

### 2.7 FINITE POPULATION MULTIPLIER

In our discussion of sampling distributions, we have used the following equation to calculate the standard error of the mean:

$$
\sigma_{\bar{x}}=\sigma / \sqrt{n}
$$

This equation is designed for situations in which the population is infinite, or in which we sample from a finite population with replacement (that is, after each item is sampled, it is put back into the population before the next item is chosen, so that the same item can possibly be chosen more than once).

In fact, many of the populations studied are finite; that is, of stated or limited size. Examples of these include the employees in a given company, the clients of a city social-services agency, the students in a specific class, and a day's production in a given manufacturing plant. So, we need to modify the equation to deal with finite populations. The formula designed to find the standard error of the mean, when the population is finite, and we sample without replacement, is

## Standard Error of the Mean for Finite Populations

$$
\sigma_{\bar{x}}=\frac{\sigma}{\sqrt{n}}\left(\sqrt{\frac{N-n}{N-1}}\right)
$$

where $N=$ size of the population
$n=$ size of the sample
The new term on the right-hand side, which we multiply by our original standard error, is called the finite population multiplier:

## Finite Population Multiplier

$$
\text { Finite population multiplier }=\sqrt{\frac{N-n}{N-1}}
$$

A few examples will help us become familiar with interpreting and using the above equation. Suppose, we are interested in a population of 20 textile companies of the same size, all of which are experiencing excessive labour turnover. Our study indicates that the standard deviation of the distribution of annual turnover is 75 employees. If we sample five of these textile companies, without replacement, and wish to compute the standard error of the mean, we would use the equation as follows:

$$
=\frac{\sigma}{\sqrt{n}}\left(\sqrt{\frac{N-n}{N-1}}\right)
$$

$$
\begin{aligned}
& =\frac{75}{\sqrt{5}}\left(\sqrt{\frac{20-5}{20-1}}\right) \\
& =(33.54)(0.888) \\
& =29.8 \text { Standard error of the mean of a finite population }
\end{aligned}
$$

In this example, a finite population multiplier of 0.888 reduced the standard error from 33.54 to 29.8 . In cases in which the population is very large in relation to the size of the sample, this finite population multiplier is close to 1 and has little effect on the calculation of the standard error. Say, that we have a population of 1,000 items and that we have taken a sample of 20 items. If we use the Equation to calculate the finite population multiplier, the result would be

$$
\begin{aligned}
\text { Finite population multiplier } & =\sqrt{\frac{N-n}{N-1}} \\
& =\sqrt{\frac{1000-20}{1000-1}} \\
& =\sqrt{\frac{980}{999}} \\
& =0.99
\end{aligned}
$$

Using this multiplier of 0.99 would have little effect on the calculation of the standard error of the mean.
This last example shows that when we sample a small fraction of the entire population (that is, when the population size $N$ is very large relative to the sample size $n$ ), the finite population multiplier takes on a value close to 1.0 . Statisticians refer to the fraction $n / N$ as the sampling fraction, because it is the fraction of the population $N$ that is contained in the sample.
When the sampling fraction is small, the standard error of the mean for finite populations is so close to the standard error of the mean for infinite populations that we might as well use the same formula for both, namely,

$$
\text { Equation } \sigma_{\bar{x}}=\sigma / \sqrt{n}
$$

The generally accepted rule is: When the sampling fraction is less than 0.05 , the finite population multiplier need not be used.

When we use the above equation, s is constant, and so the measure of sampling precision $\mathrm{s} x$ depends only on the sample size $n$ and not on the proportion of the population sampled. That is, to make $s x$ smaller, it is necessary only to make $n$ larger. Thus, it turns out that it is the absolute size of the sample that determines sampling precision, not the fraction of the population sampled.

Although the law of diminishing return comes from economics, it has a definite place in statistics too. It says that there is diminishing return in sampling. Although sampling more items will decrease the standard error, (the standard deviation of the distribution of sample means) the increased precision
may not be worth the cost. Because $n$ is in the denominator, when we increase it (take larger samples) the standard error $s x$ decreases. In our example, when we increased the sample size from 10 to 100 (a tenfold increase) the standard error fell only from 31.63 to 10 (about a two-thirds decrease). Maybe it wasn't smart to spend so much money increasing the sample size to get this result. That's exactly why statisticians (and smart managers) focus on the concept of the "right" sample size. Some finite populations are so large that they are treated as if they were infinite. An example of this would be the number of TV households in our country.

## Keywords/Glossary

Census: The measurement or examination of every element in the population.
Sample: A portion of the elements in a population chosen for direct examination or measurement.
Strata: Groups within a population formed in such a way that each group is relatively homogeneous, but wider variability exists among the separate groups.
Clusters: Groups, in population, that are similar to each other, although the groups themselves have wide internal variation.

Random or probability sampling: A method of selecting a sample from a population in which all the items in the population have an equal chance of being chosen in the sample.

Stratified sampling: It is a method of random sampling. The population is divided into homogeneous groups or strata. Elements within each stratum are selected randomly according to one of two rules.

1. A specified number of elements is drawn from each stratum corresponding to the proportion of that stratum in the population.
2. Equal numbers of elements are drawn from each stratum, and the results are weighted according to the stratum's proportion of the total population.

Systematic sampling: A method of sampling in which elements to be sampled are selected from the population at a uniform interval measured in time, order or space.
Cluster sampling: A method of random sampling. The population is divided into groups or clusters of elements, and then a random sample of these clusters is selected.

Judgment sampling: It is a method of selecting a sample from a population in which personal knowledge or expertise is used to identify the items from the population that are to be included in the sample.

Statistic: Measures describing the characteristics of a sample.
Parameters: Values that describe the characteristics of a population.
Sampling distribution of the mean: A probability distribution of the means of all the possible samples of a given size, $n$, from a population.

Sampling distribution of a statistic: For a given population, a probability distribution of all the possible values a statistic may take on for a given sample size.

Sampling error: Error or variation among sample statistic. Differences between each sample and the population and among several samples, which are due to the elements we happen to choose for the sample.
Standard error: The standard deviation of the sampling distribution of a statistic.
Standard error of the mean: The standard deviation of the sampling distribution of the mean, a measurement the extent to which we expect the means from different samples to vary from the population mean, owing to the chance error in the sampling process.
Statistical inference: The process of making inferences about populations from information contained in samples.
Central Limit Theorem: The theorem states that the sampling distribution of the mean approaches normality as the sample size increases, regardless of the shape of the population distribution from which the sample is selected.
Finite population: A population having a stated or limited size.
Finite population multiplier: A factor used to correct the standard error of the mean for studying a population of a finite size that is small concerning the size of the sample.
Infinite population: A population in which it is theoretically impossible to observe all the elements.
Sampling with replacement: A sampling procedure in which sampled items are returned to the population after being picked so that some members of the population can appear in the sample more than once.
Sampling without replacement: A sampling procedure in which sampled items are not returned to the population after being picked so that no member of the population can appear in the sample more than once.

## Equations Introduced in the Unit

## Equation 1

$$
\sigma_{\bar{x}}=\sigma / \sqrt{n}
$$

Use this formula to derive the standard error of the mean when the population is infinite, that is, when the elements of the population cannot be enumerated in a reasonable period, or when we sample with replacement. This equation states that the sampling distribution has a standard deviation, which we also call a standard error, equal to the population standard deviation divided by the square root of the sample size.

## Equation 2

$$
z=(\bar{x}-\mu) / \sigma_{\bar{x}}
$$

A modified version of the equation allows us to determine the distance of the sample mean $x$ from the population mean $\mu$, when we divide the difference by the standard error of the mean $\mathrm{s}_{x^{\prime}}$. Once we have derived a $z$ value, we can use the Standard Normal Probability Distribution Table and compute the probability that the sample mean will be that distance from the population mean. Because of the central limit theorem, we can use this formula for non-normal distributions if the sample size is at least 30 .

## Equation 3

$$
\sigma_{\bar{x}}=\frac{\sigma}{\sqrt{n}}\left(\sqrt{\frac{N-n}{N-1}}\right)
$$

where $N=$ size of the population; $n=$ size of the sample

This is the formula for finding the standard error of the mean when the population is finite, that is, of stated or limited size, and the sampling is done without replacement.

## Equation 4 <br> $$
\text { Finite population multiplier }=\sqrt{\frac{N-n}{N-1}}
$$

In Equation 3, the term $=\sqrt{\frac{N-n}{N-1}}$, which we multiply by the standard error from Equation (1), is called the finite population multiplier. When the population is small in relation to the size of the sample, the finite population multiplier reduces the size of the standard error. Any decrease in the standard error increases the precision with which the sample mean can be used to estimate the population mean.

## Questions

1. Explain Random Numbers.
2. What is a Sampling distribution?
3. What is sample mean?
4. Explain the Distribution of all sample means.
5. Define Standard Error.
6. We have a population of 10,000 , and we wish to sample 20 randomly. Use a random number table to choose the sample.
7. A population comprises groups that have wide variations within the group and less variation from group to group. Which is the appropriate type of sampling method?
8. Explain: Sampling allows us to be cost-effective. We have to be careful in choosing representative samples.
9. Suppose you are sampling from a population with a mean of 5.3. What sample size will guarantee that
(a) The sample mean is 5.3 ?
(b) The standard error of the mean is zero?
10. In a sample of 16 observations from a normal distribution with a mean of 150 and a variance of 256, what is
(a) $P(\bar{x}<160)$ ?
(b) $P(\bar{x}>142)$ ?

If, instead of 16 observations, 9 observations are taken, find
(c) $P(\bar{x}<160)$ ?
(d) $P(\bar{x}>142)$ ?
11. In a sample of 19 observations from a normal distribution with mean 18 and standard deviation 4.8
(a) What is $P(16<\bar{x}<20)$ ?
(b) What is $P(16 £ \bar{x} £ 20)$ ?
(c) Suppose the sample size is 48 . What is the new probability in part(a)?
12. In a normal distribution with a mean of 56 and standard deviation of 21 , how large a sample must be taken so that there will be at least a 90 per cent chance that its mean is greater than 52 ?
13. In a normal distribution with a mean of 375 and a standard deviation of 48 , how large a sample must be taken so that there will be at least a 0.95 probability that the sample mean falls between 370 and 380 ?
14. The average cost of a flat at Powai lake is Rs. 62 lakh, and the standard deviation is Rs. 4.2 lakh. What is the probability that a flat at this location will cost at least Rs. 65 lakh?
15. State whether the following statements are true or false.
(a) When the items included in a sample are based on the judgement of the individual conducting the sample, the sample is said to be non-random.

True/False
(b) A statistic is a characteristic of a population.

True/False
(c) A sampling plan that selects members from a population at uniform intervals in time order or space is called stratified sampling.

True/False
(d) As a general rule, it is not necessary to include a finite population multiplier in computation for the standard error of the mean when the sample size is greater than 50. True/False
(e) The probability distribution of means of all the possible samples is known as the sample distribution of the mean.

True/False
(f) The principles of simple random sampling are the theoretical foundation for statistical inference.

True/False
(g) The standard error of the mean is the standard deviation of the distribution of sample means.

True/False
(h) A sampling plan that divides the population into well-defined groups from which random samples are drawn is known as cluster sampling.

True/False
(i) With increasing sample size, the sampling distribution of the mean approaches normality, regardless of the distribution of the population.

True/False
(j) The standard error of the mean decreases with an increase in sample size. True/False
(k) To perform a complete enumeration, one would need to examine every item in a population.

True/False
(1) In everyday life, we see many examples of infinite populations of physical objects. True/False
(m) To obtain a theoretical sampling distribution, we consider all the samples of a given size. True/False
(n) Large samples are always a good idea because they decrease the standard error. True/False
(o) If the mean for a certain population were 15 , most of the samples we could take from that population would likely have a mean of 15 . True/False
(p) The standard error of a sample statistic is the standard deviation of its sampling distribution.

True/False
(q) Judgement sampling has the disadvantage that it may lose some representativeness of a sample.

True/False
(r) The sampling fraction compares the size of a sample to the size of the population. True/False
(s) Any sampling distribution can be totally described by its mean and standard deviation.

True/False
(t) The precision with which the sample mean can estimate the population mean decreases as the standard error increases.

True/False

## UNIT Measures of Central Tendency \& Dispersion, Skewness, Kurtosis

STRUCTURE
3.0 Objectives
3.1 Introduction to Measures of Central Tendency
3.2 Arithmetic Mean
3.3 Combined Arithmetic Mean
3.4 Geometric Mean
3.5 Harmonic Mean
3.6 Median and Quartiles
3.7 Mode
3.8 Introduction to Measures of Dispersion
3.9 Range and Coefficient of Range
3.10 Quartile Deviation and Coefficient of Quartile Deviation
3.11 Standard Deviation and Coefficient of Variation
3.12 Skewness and Kurtosis
Keywords
Questions

